

Unit-1 Gaseous State, Lecture-3, 1. Relation between Kinetic Energy and Temperature

Relation between Kinetic Energy & Temperature

According to Kinetic gas equation, we have

$$P = \frac{1}{3} m n \bar{c}^2$$

Let $n = N_0 =$ Avogadro's no. $= 6.022 \times 10^{23}$

$m N_0 = M =$ Molar mass = molecular mass of gas expressed in gms.

where

- $P =$ Pressure exerted by gas
- $V =$ volume of the container
- $m =$ mass of 1 molecule of gas
- $n =$ no. of molecules
- $\bar{c} =$ Root Mean square velocity

$$\therefore PV = \frac{1}{3} m N_0 \bar{c}^2$$

for 1 mole of gas molecules

$$PV = \frac{1}{3} M \bar{c}^2$$

$$\Rightarrow \frac{3}{2} PV = \frac{1}{2} M \bar{c}^2$$

$\Rightarrow 3PV = M \bar{c}^2$
multiplying both sides with $1/2$

$$\frac{1}{2} M \bar{c}^2 = KE_{av} = \frac{3}{2} PV$$

for 1 mole

$$KE_{av} \text{ for mole} = \frac{3}{2} RT$$

$$KE_{av} \propto T \text{ K}$$

for 1 mole

$$KE_{av} = \frac{1}{2} M \bar{c}^2$$

Ideal gas eqn for 1 mole
 $PV = RT$

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$$KE_{av} = \frac{3}{2} RT$$
 for 1 mole of gas molecules

$$\therefore KE_{av}/\text{molecule} = \frac{3}{2} \frac{RT}{N_0}$$

$$KE_{av}/\text{molecule} = \frac{3}{2} kT$$

$$1 \text{ mole} = 6.022 \times 10^{23} \text{ molecules} = N_0$$

$$\Rightarrow \frac{R}{N_0} = k = \text{Boltzmann const.}$$

Av. kinetic energy of gas molecules is directly proportional to temperature in Kelvin.

Derivation of Gas Laws using kinetic gas equation

① Derivation of Boyle's Law:

Statement: At constant temperature & no. of moles, the volume of gas is inversely proportional to the pressure exerted.

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Mathematically, at $T = \text{const}$, $n = \text{no. of mol} = \text{const}$

then $V \propto \frac{1}{P}$ or, $PV = K = \text{constant}$

we have the kinetic gas equation $PV = \frac{1}{3} m n \bar{c}^2$

$$= \frac{2}{3} \times \frac{1}{2} m n \bar{c}^2$$

$$= \frac{2}{3} \times \text{KE}_{\text{av}} \text{ for } n \text{ molecules}$$

And, we know that at $T = \text{const}$

$$\text{KE}_{\text{av}} \text{ for } n \text{ molecules} = \text{const}$$

$$\therefore PV = \frac{2}{3} \times \text{const}$$

$$\Rightarrow \boxed{PV = \text{const}}$$

when $n = \text{const}$
 $T = \text{const}$

which is Boyle's Law

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② Derivation of Charles's Law:

Statement

At constant Temperature & no. of molecules, the volume of gas is directly proportional to the absolute temperature.

Mathematically, at $P = \text{const}$, $n = \text{no. of molecules} = \text{const}$

$$V \propto T_K \Rightarrow \boxed{\frac{V}{T} = \text{const}}$$

According to kinetic gas equation

$$PV = \frac{1}{3} mn\bar{c}^2$$

$$= \frac{2}{3} \times \left(\frac{1}{2} mn\bar{c}^2 \right) = \frac{2}{3} \times \text{KE}_{\text{av}} = \frac{2}{3} \times \frac{3}{2} RT$$

③ Gay Lussac's Law
 $P \propto T_K$

$$P = \frac{F}{A} \times T$$

When $V = \text{const}$
 $\Rightarrow P \propto T$

$$\Rightarrow \left\{ \begin{array}{l} PV = RT \\ V = \left(\frac{RT}{P} \right) \end{array} \right.$$

at $P = \text{const}$, $\frac{R}{P} = \text{const}$
 $\therefore V \propto T_K$ which is the Charles's Law

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③ Derivation of Gay Lussac Law

statement: At constant volume & no. of moles, the pressure exerted by the gas is directly proportional to the absolute scale of temperature.

mathematically, at $V = \text{const}$, $n = \text{no. of moles} = \text{const}$.
 $\therefore P \propto T_K \Rightarrow \boxed{\frac{P}{T} = \text{const}}$

According to kinetic gas equation.

$$PV = \frac{1}{3} mn\bar{c}^2 = \frac{2}{3} \times \left(\frac{1}{2} mn\bar{c}^2 \right) = \frac{2}{3} \times KE_{av}$$

$$PV = \frac{2}{3} \times \frac{3}{2} RT = RT$$

$$\Rightarrow P = \frac{R}{V} T \quad \text{at } V = \text{const} \Rightarrow \frac{P}{V} = \text{const}$$

$\therefore \boxed{P \propto T}$ which is the Gay Lussac Law.

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④ Derivation of Avogadro's Law

Statement - \therefore Under equal conditions of temperature & pressure the volume of gas is directly proportional to the no. of moles n .

In other words \therefore Equal volume of two gases under equal conditions of temperature & pressure contain equal no. of moles.

Mathematically

$$P = \text{const}, T = \text{const}$$

$$V \propto n \Rightarrow$$

$$\frac{V}{n} = \text{const}$$

According to kinetic gas equation.

$$PV = \frac{1}{3} mn\bar{c}^2$$

$$\text{Let } n = N_1 \text{ moles}$$

$$= \frac{2}{3} \times \left(\frac{1}{2} m\bar{c}^2\right) \times N_1$$

$$= \frac{2}{3} \times (KE_{av}) \times N_1 = \frac{2}{3} \times \frac{3}{2} RT \times N_1$$

$$\therefore \text{at } T = \text{const}, \text{ at } P = \text{const} \Rightarrow \frac{RT}{P} = \text{const}$$

$$\therefore PV = RT \times N_1 \Rightarrow \frac{V}{N_1} = \frac{RT}{P}$$

$$\Rightarrow \frac{V}{N_1} = \text{const} \therefore V \propto N_1 \text{ moles}$$

which is Avogadro's Law